

LOGISTIC
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The 'logistic' difference function $P_{n+1} \leftarrow r \cdot P_n \cdot (1 - P_n)$ is the simplest descriptor for population growth. Successive iterations of this function with various initial populations (P) in the range 0 to 1 and various rates of population growth in the range 0 to 4 under ideal conditions describe populations at generation n.

For example, If one starts with a population of 0.1 and an birthrate of 2, we get the following sequence:

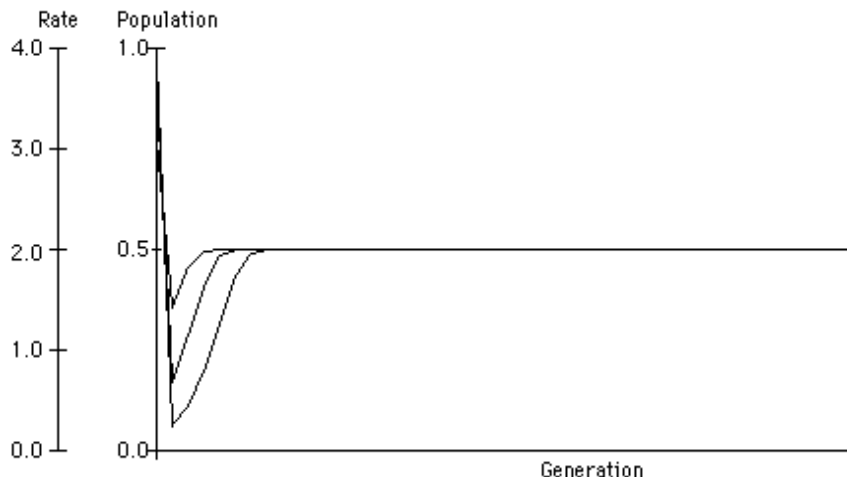
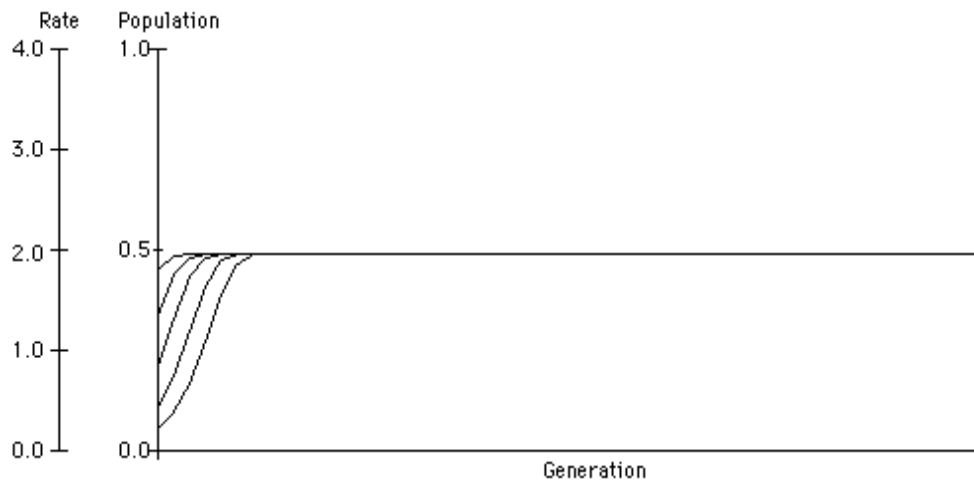
$$P_1 \leftarrow 2 \cdot 0.1 \cdot (1 - 0.1) = 0.18$$

$$P_2 \leftarrow 2 \cdot 0.18 \cdot (1 - 0.18) = 0.324$$

etc.

Run the program. You see a graph with population on the vertical axis and generations on the horizontal axis. Move the cursor up and down along the vertical axis and observe how the initial population value is altered at the top of the screen. Runing the cursor up and down the Rate axis allows you to set the population birthrate, also indicated at the top of the screen.

Set the birthrate to 2.0 and with various initial populations in the range 0 to 0.5, press x to activate the calculation process. **These conditions produce increasing populations that rise rapidly to the limiting value of 0.5**

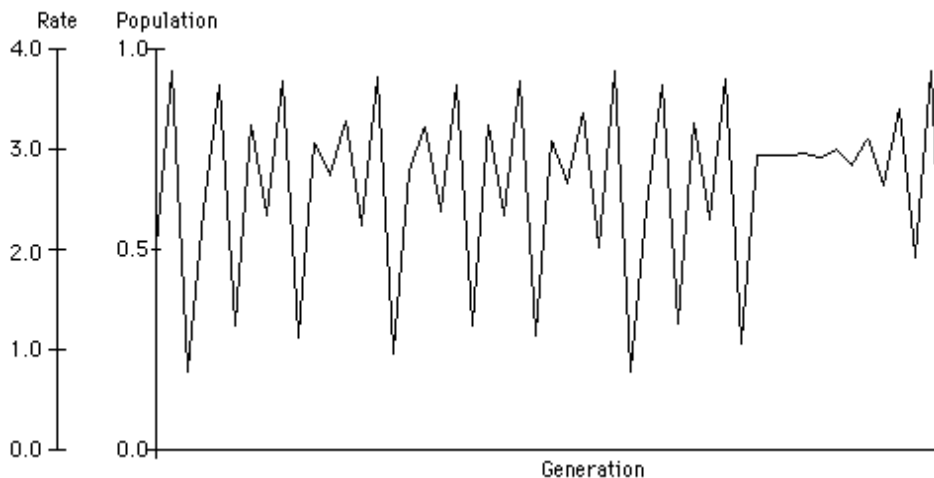
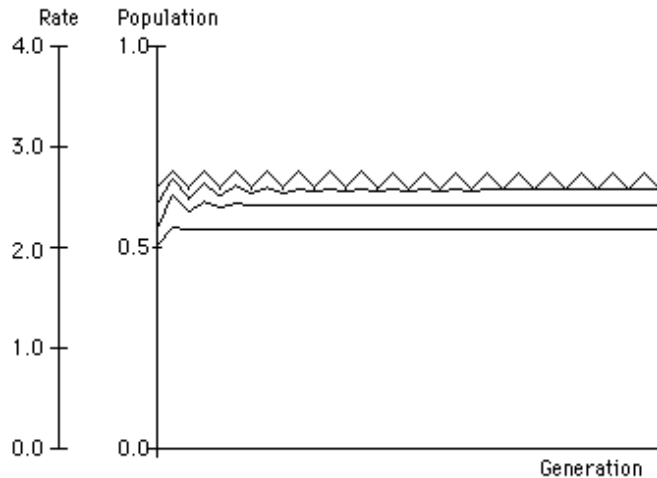


Continuing with the birthrate at 2.0, activate the calculations with various initial populations in the range 0.5 - 1.0 **These also stabilize at 0.5; but there is an initial dip below this value.**

Experiment with birthrate set to 1.0 **Populations with birthrate less than 2.0 decrease. Dramatically at first, but then levelling off and 'petering out'.**

. WE WILL IN ALL SUBSEQUENT EXPERIMENTATION LEAVE THE INITIAL POPULATION AT 0.5 AND ONLY VARY THE BIRTHRATE.

At various birthrates slightly in excess of 2.0, **populations fluctuate... at lower birthrates in dampened way, at higher rates in an undampened way.**



It is possible to 'skip the first 100 and first 1000 calculations so that we can see what happens 'down the line'.

Slowly increasing the birthrate allows us to observe that **at some point there is period doubling** which means that **populations fluctuate between four values**.

Press the <r> key.

You are asked to input an initial value for birthrate. Enter 0 (zero). You are also asked to provide an ending value. Enter 4. For resolution enter 10. Press the <x> key.

The resulting graph shows that **populations are not sustainable with birthrates less than 1. As birthrates increase there is an increase in the stable population. At some point the populations in succeeding generations fluctuate between two values. This is called period doubling or bifurcation. Further birthrate increases lead to further doubling. Eventually there is chaos.**

By pressing <r> and running this simulation again with different values for starting and ending r and resolution you can 'zoom in' on selected portions of the bifurcating and chaotic regions. You will discover **patterns in chaos**. You will also discover that **stability 'crystalizes' out of chaos at certain values for r** .

The screen displays produced by this program can be 'dumped' either to a MacPaint file on the startup disk or to the selected Imagewriter printer by pressing <command>/<shift>/<3> or <command>/<shift>/<4>. You may want to do this and write up your own illustrated explanation for the behavior of the logistic function.

